Adaptive frequency oscillators and applications

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***Abstract*— In this contribution we present a generic mechanism to transform an oscillator into an adaptive frequency oscillator, which can then dynamically adapt its parameters to learn the frequency of any periodic driving signal. Adaptation is done in a dynamic way: it is part of the dynamical system and not an offline process. This mechanism goes beyond entrainment since it works for any initial frequencies and the learned frequency stays encoded in the system even if the driving signal disappears. Interestingly, this mechanism can easily be applied to a large class of oscillators from harmonic oscillators to relaxation types and strange attractors. Several practical applications of this mechanism are then presented, ranging from adaptive control of compliant robots to frequency analysis of signals and construction of limit cycles of arbitrary shape.**

I. Introduction

Nonlinear oscillators are very important modeling tools in biological and physical sciences and these models have gained a particular attention in engineering fields over the last decades. These models are interesting because of their synchronization capabilities with other oscillators or with external driving signals. However their synchronization capabilities are limited and it is not always an easy task to correctly choose their parameters to ensure proper synchronization with external driving signals. Indeed, an oscillator has a finite entrainment region that depends on many parameters, such as coupling strength and frequency difference between the oscillator and the driving signal.

Some recent work, however, showed that it is possible to modify oscillators such that they can overcome these synchronization limitations, by adding some dynamics to the parameters of the oscillator such that it can learn the frequency of an input signal. But these attempts are often limited to simple classes of oscillators, equivalent to phase oscillators [1], [2] or to simple classes of driving signal (pulses) [3].

Recently we designed a learning mechanism for oscillators which adapts their frequency to the frequency of any periodic input signal [4], [5]. The parameter with the strongest influence on the frequency of the oscillator is turned into a new state variable of the system. Interestingly this mechanism appears to be generic enough to be applied to many different types of oscillators, from phase oscillators to relaxation types, and to strange attractors. The frequency adaptation process goes beyond mere entrainment because if the input signal disappears, the learned frequency stays encoded in the oscillator. Moreover, it is independent of the initial conditions, thus working beyond entrainment basins (i.e. it has an infinite basin of attraction). We called this adaptation mechanism *dynamic Hebbian learning* because it shares similarities with correlation-based learning observed in neural networks [6].

In this contribution, we present this generic adaptation mechanism. Then we show several applications, ranging from adaptive control for legged robots with passive dynamics [4], [7], where the adaptive oscillators find the resonant frequency of the robot to frequency analysis with systems of coupled adaptive oscillators [8] and construction of limit cycles with arbitrary shape [9].

II. Adaptive frequency oscillators

A. A generic rule for frequency adaptation

We consider general equations for an oscillator perturbed by a periodic driving signal

$$\begin{aligned}\dot{x} &= f_x(x, y, \omega) + KF(t)\\ \dot{y} &= f_y(x, y, \omega)\end{aligned}$$

where f_x and f_y are functions of the state variables that produce a structurally stable limit cycle and of a parameter ω that has a monotonic relation with the frequency of the oscillator when unperturbed, $K = 0$ (we do not require this relation to be linear). $F(t)$ is a time periodic perturbation and $K > 0$ the coupling strength.

In order to make the oscillator learn the frequency of $F(t)$, we transform the ω parameter into a new state variable, that will have its own dynamics. The generic rule that allows us to transform this oscillator into an adaptive frequency one is as follows

$$\dot{\omega} = \pm KF(t)\frac{y}{\sqrt{x^2 + y^2}}$$

where the sign depends on the direction of rotation of the limit cycle in the (x, y) plane.

B. Properties of the adaptation mechanism

We proved in [5] that the adaptation mechanisms made the frequency converge to the frequency of any periodic input signal for phase and Hopf oscillators. In the case of several frequencies in the spectrum of $F(t)$, the frequency converges to one of the frequency component, depending on the initial frequency of the oscillator.

Moreover, the higher the coupling strength K is, the faster the convergence. It can be shown that for suitable coupling strength, the convergence is exponential (of order e^{-t}). Examples of frequency adaptation for the Hopf oscillator, with several different inputs is shown in Figure 1. The corresponding equations for the adaptive Hopf oscillator are

$$\begin{aligned}\dot{x} &= (\mu - x^2 - y^2)x - \omega y + KF(t)\\ \dot{y} &= (\mu - x^2 - y^2)y + \omega x\\ \dot{\omega} &= -KF(t)\frac{y}{\sqrt{x^2 + y^2}}\end{aligned}$$

We can notice in Figure 1(d) that the adaptation mechanism works also for time-varying signals (i.e. with a time-varying frequencies). This tracking is however limited by the exponential convergence rate of the adaptation mechanism. Further examples of such tracking and limitations can be found in [8] for pools of oscillators.

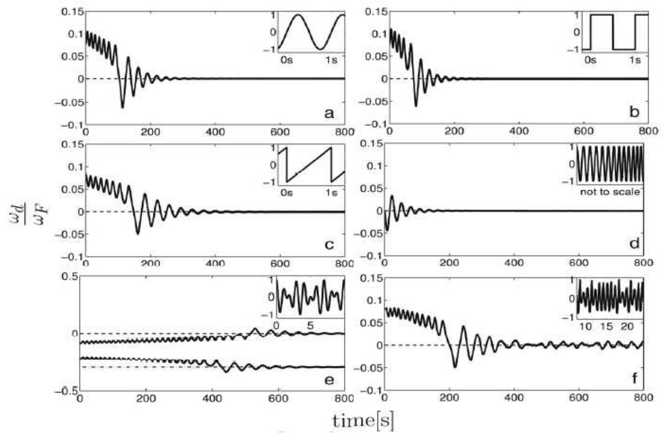
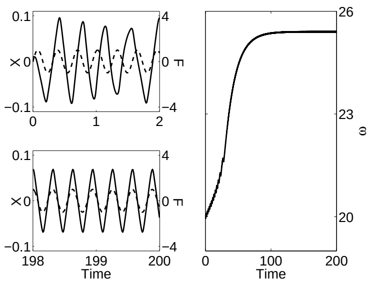


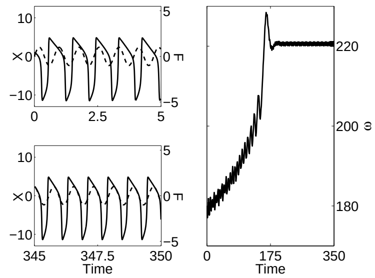
Fig. 1. (a) Typical convergence of an adaptive frequency Hopf oscillator driven by a harmonic signal ($(F(t) = \sin(2\pi t)$). The frequencies converge towards the frequency of the input (indicated in dashed line). After convergence the frequency oscillates with a small amplitude around the frequency of the input. In all figures, we plot in the main graph the time evolution of the difference between ω and the input frequency, normalized by the input frequency. The top right panel shows the driving signals (note the different scales). (b) Square pulse $I(t) = \mathrm{rect}(\omega_F t)$, (c) Sawtooth, $I(t) = \mathrm{st}(\omega_F t)$, (d) Chirp $I(t) = \cos(\omega_c t)$, where $\omega_c = \omega_F(1 + \frac{1}{2}(\frac{t}{1000})^2)$. (Note that the graph of the input signal is illustrative only since changes in frequency takes much longer than illustrated). (e) Signal with two non-commensurate frequencies $I(t) = \frac{1}{2}\left[\cos(\omega_F t) + \cos(\frac{\sqrt{2}}{2}\omega_F t)\right]$, i.e. a representative example how the system can evolve to different frequency components of the driving signal depending on the initial condition $\omega_d(0) = \omega(0) - \omega_F$. (f) $I(t)$ is the non-periodic output of the Rössler system. The Rössler signal has a $1/f$ broad-band spectrum, yet it has a clear maximum in the frequency spectrum. In order to assess the convergence we use $\omega_F = 2\pi f_{\max}$, where $f_{\max}$ is found numerically by FFT. The oscillator converges to this frequency.

Moreover, extensive numerical simulations show that this adaptation mechanism works also for many different types of non-harmonic oscillators. Some examples are shown in Figure 2, with an adaptive Rayleigh oscillator, an adaptive Fitzhugh-Nagumo one and a Rössler system in chaotic mode. For the first two oscillators there is no linear relation between ω and the frequency of oscillations but the adaptive mechanism is able to find a suitable value for ω such that the frequency of the oscillator is the same as the frequency of the input signal. For the Rössler system, the frequency of the system is not well defined since the system is not periodic, but we can define a pseudo-frequency and the system can adapt it to the frequency of a periodic input.



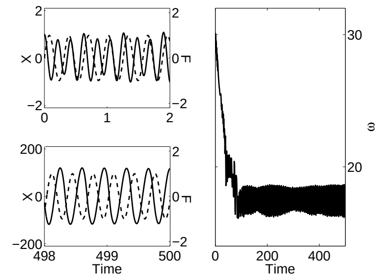
$$
\begin{aligned}
\dot{x} &= y + KF \\
\dot{y} &= \delta(1 - qy^2)y - \omega^2 x \\
\dot{\omega} &= KF \frac{y}{\sqrt{x^2+y^2}}
\end{aligned}
$$

(a) Adaptive Rayleigh oscillator



$$
\begin{aligned}
\dot{x} &= x(x-a)(1-x) - y + KF \\
\dot{y} &= \omega(x - by) \\
\dot{\omega} &= -KF \frac{y}{\sqrt{x^2+y^2}}
\end{aligned}
$$

(b) Adaptive Fitzhugh-Nagumo oscillator



$$
\begin{aligned}
\dot{x} &= -\omega y - z + KF \\
\dot{y} &= \omega x + ay \\
\dot{z} &= b - cz + xz \\
\dot{\omega} &= -KF \frac{y}{\sqrt{x^2+y^2}}
\end{aligned}
$$

(c) Adaptive Rössler system

Fig. 2. For each oscillator, ω corresponds to the adaptive parameter. Each figure is composed of 3 plots. The right one shows the evolution of ω. The left ones are plots of the oscillations (the x variable) and of the input signal F (dashed line), before (upper figure) and after (lower figure) adaptation.

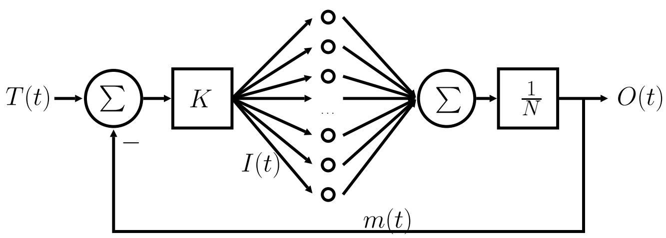


Fig. 3. Structure of the pool of adaptive frequency oscillators that is able to reproduce a given teaching signal $T(t)$. The mean field produced by the oscillators is fed back negatively on the oscillators.

III. Applications

We now present several applications for the adaptation mechanism, ranging from robot control to frequency analysis and automatic construction of limit cycles of arbitrary shape.

A. Robot with passive dynamics

The adaptation mechanism can be used to find the resonant frequencies of legged robots with passive elements (i.e. springs) [4], [7], [10]. The developed controller, based on adaptive frequency oscillators, can tune itself to the resonant frequency of the robot via a simple feedback loop from sensors on the robot (e.g. position or inertial sensors). The locomotion thus becomes very efficient by exploiting the intrinsic dynamics of the robot. Another advantage of this type of control is that one does not need to tune the controller to specific robot and the controller can track any change in this frequency automatically, for example if this frequency change due to mass or spring stiffness changes or to a gait transition (if the robot is stepping on 2 feet or 4 feet, its resonant frequency changes).

B. Frequency analysis

Another application is the use of a pool of adaptive frequency Hopf oscillators to perform frequency analysis of an input signal [8]. The oscillators are coupled via a negative mean field with the input signal to analyze as is shown in Figure 3. The oscillators converge to the frequencies present in the spectrum of the teaching signal and due to the negative feedback, each time an oscillator finds a correct frequency, this one loses its amplitude. Thus, the other oscillators only *feel* the remaining frequencies to learn.

This pool of oscillators is able to approximate the frequency spectrum of any signal. This works for signals with discrete spectra, but also for continuous spectra and time-varying spectra. The spectrum is approximated by the distribution of the frequencies of the oscillators and thus, the resolution of the approximation can be made arbitrary good by increasing the number of oscillators present in the pool.

Figure 4 shows how the system can approximate the spectrum of a broad-band chaotic signal from the Rössler system. As can be seen, the important features of the spectrum are caught by the system, especially the broad spectrum and the major peaks of frequency.

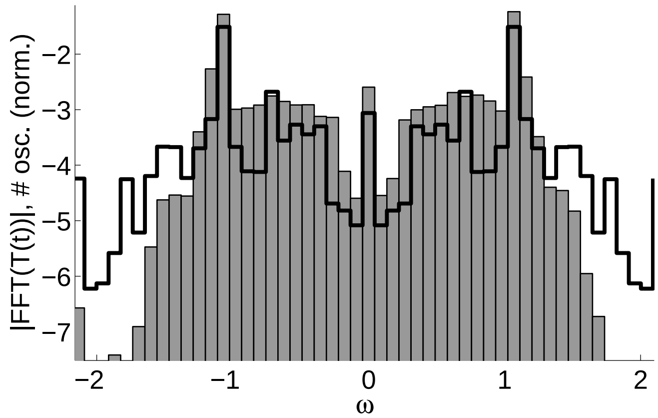


Fig. 4. FFT of the Rössler signal (black line) in comparison with the distribution of the frequencies of the oscillators (gray bars normalized to the number of oscillators, $N = 10000$). The spectrum of the signal has been discretized into the same bins as the statistics of the oscillators in order to allow for a good comparison with the results from the full-scale simulation.

C. Construction of limit cycles with arbitrary shape

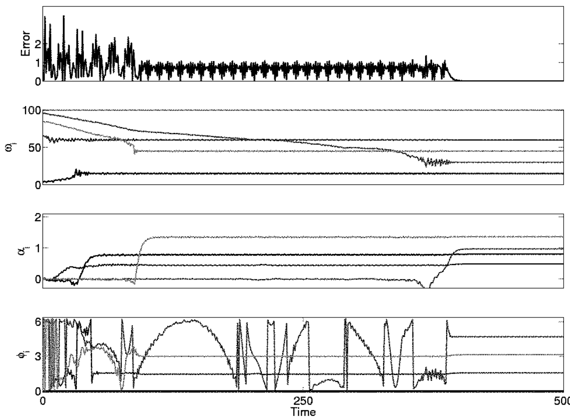
The previous pool of oscillators can be extended by adding a weight to each oscillator in the mean field sum and coupling between the oscillators to ensure stability of the constructed pattern. Then an oscillator will be able to fully match the energetic content of a frequency in the spectrum of the teaching signal. Moreover the coupling will ensure that the system exhibits a stable limit cycle. The amplitudes and phase differences become also state variables of the system as for the frequencies. The governing differential equations of the system are then

$$\begin{aligned}
\dot{x}_i &= (\mu - r_i^2)x_i - \omega_i y_i + KF(t) + \tau \sin(\theta_i - \phi_i) \\
\dot{y}_i &= (\mu - r_i^2)y_i + \omega_i x_i \\
\dot{\omega}_i &= -KF(t)\frac{y_i}{r_i} \\
\dot{\alpha}_i &= \eta x_i F(t) \\
\dot{\phi}_i &= \sin\left(\frac{\omega_i}{\omega_0}\theta_0 - \theta_i - \phi_i\right)
\end{aligned}$$

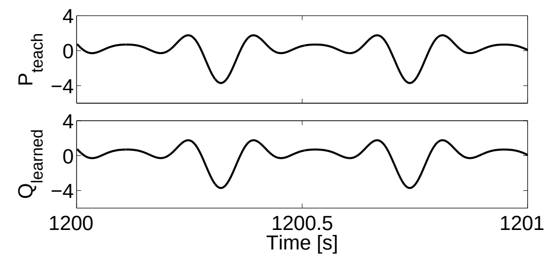
with

$$\begin{aligned}
\theta_i &= \operatorname{sgn}(x_i)\cos^{-1}\left(-\frac{y_i}{r_i}\right) \\
F(t) &= P_{\text{teach}}(t) - Q_{\text{learned}}(t) \\
Q_{\text{learned}}(t) &= \sum_{i=0}^{N} \alpha_i x_i
\end{aligned}$$

where τ, K and η are positive constants. The output of the system, Q_{learned}, is the weighted sum of the outputs of each oscillator. $F(t)$ represents the negative feedback, which in average is the remaining of the teaching signal $P_{\text{teach}}(t)$ the network still has to learn. α_i represents the amplitude associated to the frequency ω_i of oscillator i. Its equation of evolution maximizes the correlation between x_i and $F(t)$, which means that α_i will increase only if ω_i has converged to a frequency component of $F(t)$ (the correlation will be positive in average) and will stop increasing when the frequency component ω_i will disappear from $F(t)$ because of the negative feedback loop. ϕ_i is the phase difference between oscillator i and 0. It converges to the phase difference between the instantaneous phase of oscillator 0, θ_0, scaled at frequency ω_i and the instantaneous phase of oscillator i, θ_i. Each adaptive oscillator is coupled with oscillator 0, with strength τ to keep correct phase relationships between oscillators.



(a) Evolution of the state variables of the system



(b) Result of learning

Fig. 5. Construction of a limit cycle by learning an input signal ($P_{\text{teach}} = 0.8\sin(15t) + \cos(30t) - 1.4\sin(45t) - 0.5\cos(60t)$). Figure 5(a) shows the evolution of the state variables of the system during learning. The upper graph is a plot of the error ($\|P_{teach} - Q_{learned}\|$). The 3 other graphs show the evolution of the frequencies, ω_i, the amplitudes, α_i and the phases, ϕ_i. We clearly see that the system can learn perfectly the teaching signal, the frequencies, amplitudes and phase differences converge to the correct values and the error become zero. Figure 5(b) shows the result of learning (teaching signal in upper graph, output of the system in lower one), we notice the perfect reconstruction of the signal.

Figure 5 shows an example of convergence of the network of oscillators with amplitudes and coupling together with the resulting learned signal. We see that the frequencies first converge to the different frequency components present in the signal, the amplitudes increase when the associated frequency matches one frequency of the input signal. At the end, the phase differences stabilize and we see that the error is zero, which means that the system perfectly reconstructed the teaching signal. Moreover, it is now encoded into a structurally stable limit cycle and it is easy to smoothly modulate its frequency and amplitude by changing $\vec{\omega}$ and $\vec{\alpha}$. These properties can be very useful, together with sensory feedback, for robotics control (see for example [9]). This system can be viewed as a dynamic Fourier series decomposition where there is not need of explicit definition of a time window or any preprocessing of the signal to analyze.

IV. Conclusion

In this contribution we presented a generic mechanism to build adaptive frequency oscillators from a given existing oscillator. We showed that it could be applied to many different types of oscillators, that the system was able to learn the frequencies of any periodic input signal. Interestingly there is no need to preprocess the signal and no external optimization procedures are needed to get the correct frequency. All the learning is embedded into the dynamics of the adaptive oscillators. Moreover it goes further than entrainment, since the learned frequency is kept into the system even if the external drive disappears and the basin of attraction is infinite (the system can start from any initial frequency). Finally we discussed some applications of this mechanism, ranging from adaptive control for compliant robots, to frequency analysis and construction of limit cycles of arbitrary shape.

Acknowledgment

This work was supported by the European Commission's Cognition Unit, project no. IST-2004-004370: RobotCub (L.R.) and by a grant from the Swiss National Science Foundation (L.R, J.B. and A.I.)

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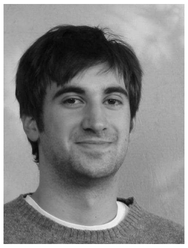
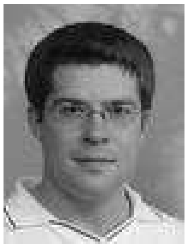


Ludovic Righetti is doing a PhD at the Biologically Inspired Robotics Group (BIRG) at the Ecole Polytechnique Fédérale de Lausanne (EPFL). He has a BS/MS in Computer Science from EPFL (March 2004). He is particularly interested in the theoretical aspects of locomotion, both in animals and legged robots, and more generally in control theory and dynamical systems.



Jonas Buchli is currently a Post-doctoral Fellow at the Computational Learning and Motor Control Lab at the University of Southern California in Los Angeles. He received a Diploma in Electrical Engineering from the ETHZ (the Swiss Federal Institute of Technology, Zürich) in 2003 and a Doctorate from the EPFL (the Swiss Federal Institute of Technology, Lausanne) in 2007. His research interests include self-organization and emergent phenomena in complex systems, the theory of nonlinear dynamical systems and information concepts. And, especially the applications of these topics to the intersecting field of engineering and biology.

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