Non-Linear Dynamics in a Small-Open-Economy Model in the Euro Area

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Abstract— **This article focuses on the dynamics of the commodity and money market. The purpose of the article is to study a two-equation linear dynamic model in order to examine its behavior in complex conditions and its dependence on the parameters. Stability conditions are examined by means of the Ruth - Hurwitz criterion. The system and its stability are also examined when non-linear dynamics is introduced. The linear system was stable for any economically admissible values of the parameters. Also, non-linear perturbations were applied in two cases which both led to a stable system. From a theoretical perspective in economics this implies that for any small open economy operating in the Euro area without exchange rate dynamics and operating under the specific conditions, regardless of the possible non-linearity in the investment function, the resulting economic system is asymptotically stable.**

Keywords— small open economy; dynamical system; non-linear; numerical.

I. Introduction

The model presented in this paper is primarily of Keynesian inspiration and is based on the Mundell-Fleming theory using IS-LM model [1], [2]. This article models a small open economy without exchange rate dynamics as a two-equation dynamic model in order to examine its behavior. This model could be suitable for modeling the economy of a country such as Greece operating in the Euro area, where exchange rates are absent. In the commodity market, demand for investment and net export is related to savings. Total demand and savings are said to be in equilibrium. The investment demand function depends on the interest rate. The net export demand is considered to be constant. First, we begin with an explanation of the linear model where the economic nature of the parameters results in stability.

II. The Model

Production Y is described by the equation:

$$Y' = a\left[I(R,Y) + X(Y,Z) - S(Y,Z)\right], a > 0 \qquad (1)$$

Equation (1) simply expresses the difference between aggregate demand and aggregate supply. Aggregate demand is equal to C + I + E where C, I, E are investment, consumption and exports, respectively, and aggregate supply is equal to C + S + M where C, S, M, are consumption, savings and imports, respectively. Subtracting aggregate supply from aggregate demand we get I + X – S where X = E – M denotes net export [3]. The difference between aggregate demand and supply causes the increase of production. If the difference is positive, this implies an increase in production and if the difference is negative this implies a decrease in production [4]. Dividing equation (1) by Y we get:

$$\frac{Y'}{Y} = a\left[\frac{I(Y,R)}{Y} + \frac{X(Y)}{Y} - \frac{S(Y)}{Y}\right] \qquad (2)$$

The investment function has the form:

$$I = (i_0 - i_2 R)Y, \quad i_0, i_2 > 0 \qquad (3)$$

The net exports function has the form:

$$X = x_0 Y, \quad x_0 > 0 \qquad (4)$$

The savings function has the form:

$$S = (s_0 + s_1 \ln Y)Y, \quad s_0, s_1 > 0 \qquad (5)$$

Also, let:

$$y = \ln Y \qquad (6)$$

$$x = \ln X \qquad (7)$$

Substituting equations (3)-(7) into equation (2) we get:

$$y' = a\left[i_0 - i_2 R + x_0 - (s_0 + s_1 y)\right] \qquad (8)$$

For the money market we assume that:

$$e^{R'} = \left[\frac{L(Y,R)}{M}\right]^{b}, b > 0 \qquad (9)$$

Or:

$$R' = b\left[l_0 + l_1 y - l_2 R - m\right] \qquad (10)$$

Where M and L denote money stock and demand, respectively.

Finally, the dependence of the demand for money on production and nominal interest rate is given by the following function:

$$L(Y,R) = l_0 \frac{Y^{l_1}}{(1+R)^{-l_2}},\ l_0, l_1, l_2 > 0 \qquad (11)$$

The final model consists of equations (8) and (10):

$$y' = a\left[i_0 - i_2 R + x_0 - (s_0 + s_1 y)\right]$$

$$R' = b\left[l_0 + l_1 y - l_2 R - m\right]$$

The model is linear and stable for each economically admissible value of the parameters.

III. Equilibrium and Stability of the Linear Model

The equilibrium of dynamical systems is defined as the situation where the variables do not evolve in time. So our system in equilibrium conditions is [5]:

$$y' = a\left[i_0 - i_2 R + x_0 - (s_0 + s_1 y)\right] = 0$$

$$R' = b\left[l_0 + l_1 y - l_2 R - m\right] = 0$$

Next we examine the stability of the system by reference to the well known Ruth – Hurwitz criterion. So, the Jacobian of the original system is as follows [6]:

$$A = \begin{bmatrix} as_1 & -ai_2 \\ bl_1 & -bl_2 \end{bmatrix}$$

The eigenvalues of the matrix are computed as roots of the following equation:

$$|A - \lambda I| = \begin{vmatrix} -as_1 - \lambda & -ai_2 \\ bl_1 & -bl_2 - \lambda \end{vmatrix} =$$

$$\lambda^2 + (as_1 + l_2 b)\lambda + ab(s_1 l_2 + i_2 l_1) = 0$$

We construct the Ruth-Hurwitz matrices:

$$H^1 = as_1 + l_2 b > 0$$

$$H^2 = \begin{vmatrix} as_1 + l_2 b & 1 \\ 0 & ab(s_1 l_2 + i_2 l_1) \end{vmatrix} > 0$$

According to the Ruth-Hurwitz rule the necessary and sufficient condition for a linear system to be asymptotically stable [$\mathrm{Re}(\lambda^i)<0$] is $H^i>0$ for all i. Given that H^1, $H^2>0$ this result implies that the system is asymptotically stable [7]. Consequently, small open economies operating in the euro area, i.e. without exchange rate dynamics are asymptotically stable.

A. *Numerical Example:*

Let us introduce a numerical example by using the following numerical values which are typically relevant for a small open economy without exchange rate dynamics:

$i_0 = 0.048, i_2 = 0.16, x_0 = 0.26, s_0 = 0.1, s_1 = 0.07, s_2 = 0.16,$ $l_0 = 0.25, l_1 = 0.12, l_2 = 0.6, M = 0.48$

The behavior of the linear system is depicted in Figs.1-2 where the evolution in time of production and interest rate are shown.

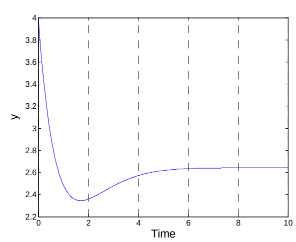

Figure 1

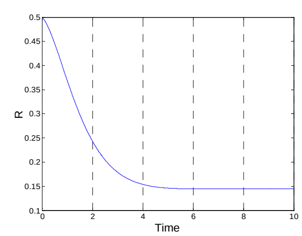

Figure 2

IV. Simple Non – Linear System

Typically, the dependence of investment on product is non-linear. If we consider a typical non-linear investment function of the logistic form:

$$I = \left(\frac{k}{1+e^{a-y}} - i_2 R\right)Y \ , \ i_2 > 0$$

Instead of (8) we get:

$$y' = a\left[\frac{k}{1+e^{a-y}} - i_2 R + x_0 - (s_0 - s_1 y)\right]$$

A. Numerical Example:

We use the same numerical values as in the previous example. Additionally, we assume that k=0.4 and a=4. The Jacobian, the determinant and the eigenvalues of the system can be computed routinely. The behavior of the non-linear system is shown in Figs. 3-4 where the evolution in time of production and interest rate are shown.

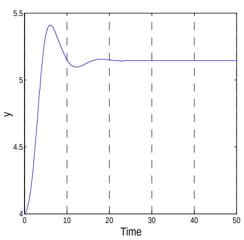

Figure 3

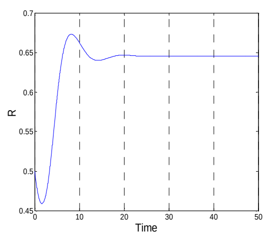

Figure 4

V. General Non – Linear System

If we consider a more general non-linear investment function of the form:

$$I = \left(\frac{1}{1+R} \cdot \frac{k}{1+e^{b-ay(t)}} - i_2 R\right)Y \ , \ i_2 > 0$$

Instead of (8) we get:

$$y' = a\left[\frac{1}{1+R} \cdot \frac{k}{1+e^{b-ay(t)}} - i_2 R + x_0 - (s_0 - s_1 y)\right]$$

A. Numerical Example:

We use the same numerical values as in the previous example. Additionally, we assume that b=1.0 and a=4. The Jacobian, the determinant and the eigenvalues of the system can be computed routinely. The behavior of the non-linear system is shown in Figs. 5-6 where the evolution in time of production and interest rate are shown.

VI. Conclusions

Microeconomic models deal with the behavior and the decisions of individuals, households and firms and the way their behavior determines prices and quantities [8]. In this context some authors have presented models of a small open economy from a microeconomic perspective based on the idea that a representative consumer chooses his equilibrium by solving inter-temporal optimization problems [9]. In this paper, we have introduced a macroeconomic model which deals with the performance and behavior of the national economy as a whole [10]. Macroeconomic models are typically used by governments and corporations in the formulation of business strategy and policy rules.

The macroeconomic linear system of the Small Open Economy model for a country operating in the Euro area without exchange rate dynamics was examined. The linear system was stable for any economically admissible values of the parameters. Also, non-linear perturbations were applied in two cases which both led to a stable system. From a theoretical perspective in economics this implies that for any small open economy operating in the Euro area without exchange rate dynamics under the specific conditions, regardless of any possible non-linearity in the investment function, the resulting economic system is stable.

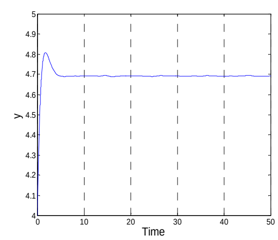

Figure 5

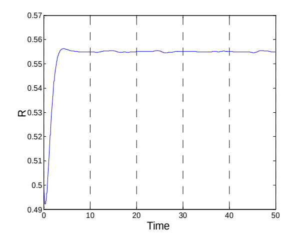

Figure 6

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