Performance Comparison of Band-limited Baseband Synchronous CDMA Using between Walsh-Hadamard Sequence and ICA Sequence

Ryo Takahashi
Next Generation Mobile Communications Laboratory,
Center for Intellectual Property Strategies, RIKEN
2–1, Hirosawa, Wako-shi, Saitama 351–0198, Japan
Email: ryo-takahashi@riken.jp

Ken Umeno
Next Generation Mobile Communications Laboratory,
Center for Intellectual Property Strategies, RIKEN
2–1, Hirosawa, Wako-shi, Saitama 351–0198, Japan
Email: chaosken_umeno@riken.jp

***Abstract*— Performance of band-limited baseband synchronous CDMA using the orthogonal Independent Component Analysis (ICA) spreading sequences is compared with that using the Walsh-Hadamard sequences. The system we focus on here is where the transmitted signal powers after passing band-limiting filter are not controlled. The orthogonal ICA sequences are generated by ICA. We use ICA not as separator for received signal but as generator of spreading sequences. These performances are also compared with that in the band-unlimited system using orthogonal sequences. We calculate Bit Error Rates (BERs) of these systems by numerical simulation. The quantities BERs in the band-limited baseband systems are lower than that in the band-unlimited baseband system. However, BER in the band-limited system using the orthogonal ICA sequences is much lower than that using the Walsh-Hadamard sequences.**

I. Introduction

In Code Division Multiple Access (CDMA), each of the signals existing in the same frequency band at the same time is assigned as the spreading sequences which are to be orthogonalized as much as possible. In the synchronous CDMA, such as the down-link, the Walsh-Hadamard sequences which realize zero interference are used.

On the other hand, Independent Component Analysis (ICA) for blind source separation attracts much attention in various fields [1], [2]. This technique makes it possible to recover the original signals from only the mixed observed signal data, without knowing how to mix original signals. The observed data $\mathbf{X}(t)$ at time t is expressed as $\mathbf{X}(t) = \mathbf{AS}(t)$, where each row of the matrix $\mathbf{S}(t)$ means the original signal and $\mathbf{A}$ is an unknown mixing matrix. Under the assumption that each original signal is independent variable and their probability distributions are nongaussian, the separating matrix $\mathbf{W}$, such that $\mathbf{S}^{\mathrm{ICA}}(t) = \mathbf{WX}(t)$, is found by maximizing the nongaussianity of the data. Here, $\mathbf{S}^{\mathrm{ICA}}(t)$ is the recovered signal data by ICA which are scaled and permuted version of the original source signals.

Recently, we have originally proposed the new orthogonal spreading sequences as the Walsh-Hadamard sequences for CDMA, which are generated by ICA [3–6]. In this proposal, ICA is used in information and communication fields not as the separator for received signal, such as Zero Forcing and Minimum Mean Square Error (MMSE) methods, but as the generator of spreading sequences. These orthogonal ICA spreading sequences can be obtained by the following simple procedure. That is to say, we apply the centering process to the original sequences, mix the sequences' data linearly and recover the sequences by ICA. The ICA sequences have almost the same waveform as the original sequences. Thus, The orthogonal ICA sequences can realize the ideal correlation property which the Walsh-Hadamard sequences do not have, although the orthogonal ICA sequences have the orthogonality like the Walsh-Hadamard sequences have.

In this paper, the performance of the band-limited baseband synchronous CDMA using the orthogonal ICA sequences is investigated and compared with that using the Walsh-Hadamard sequence. In [4], [5], the performances of these systems are investigated in the case that the frequency band is not limited. In that case, it is found that Bit Error Rate (BER) in these system using the orthogonal ICA sequence is almost the same as that using the Walsh-Hadamard sequence. In the practical CDMA system, however, the frequency bandwidth is limited. For this, the orthogonality between the simultaneous signals in the synchronous CDMA, which is realized by using the orthogonal spreading sequences, are broken. In addition, a certain amount of bit energy is cut by a filter introduced for limiting transmitted signals' bandwidth. In this study, we investigate the system where the transmitted signal powers after passing the filter are not controlled. Thus, we investigate the performance in the band-limited CDMA using the orthogonal ICA sequence and the Walsh-Hadamard sequences by numerical method. For limiting the frequency f bandwidth, we use a Raised Cosine filter whose impulse response $H(f)$ is given as

$$H(f) = \begin{cases} 1, & 0 \le |f| < \frac{1-\alpha}{2T} \\ \cos^2\left[\frac{\pi T}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right], & \frac{1-\alpha}{2T} \le |f| < \frac{1+\alpha}{2T} \\ 0, & \text{otherwise} \end{cases}$$

where α is a roll-off factor. In this study, we set $\alpha = 0.22$. By using this filter, the bandwidth is limited as $(1+\alpha)/T$.

II. Orthogonal ICA Sequence

In this paper, we consider the data recovered by ICA as the spreading sequences, namely ICA sequence. Here, we use FastICA [7] which is one of the ICA algorithms. In brief, the FastICA takes the following steps for recovering the original sequences from the observed data. First, each of the mean values of data is removed from the observed data, namely centering $E\{\mathbf{X}\} = 0$, and be whitened $E\{\mathbf{X}\mathbf{X}^{\mathrm{T}}\} = \mathbf{I}$, as preprocessing. Here, $\mathbf{I}$ means the identity matrix. Next, the mixing matrix $\mathbf{W}$ is guessed by maximizing the property of the nongaussian of these data. Finally, we can obtain K spreading sequences which length are N, but there are restriction such as $K < N$. Here, we use the program of the FastICA in IT++ [8] which is a C++ library in the numerical simulation.

The ICA sequence have several unique properties compared with the original sequences although the waveforms of the recovered ones are almost the same as those of original ones [3], [4]. Because of using this ICA algorithm, the sequences recovered by ICA, namely $\{S^{\mathrm{ICA}}_{k,j}\}$, satisfy the following relations:

$$S^{\mathrm{ICA}}_{k,j} \simeq S_{k,j}/D_{S_k}, \quad (1)$$

$$S^{\mathrm{ICA}}_{k,j} = S'_{k,j} + \overline{S_k}/D_{S_k}, \quad (2)$$

where $\overline{S_k}$ and D_{S_k} mean the length of the sequence N average and the standard deviation of $\{S_{k,j}\}$, respectively. Because of using the centering and the whitening in the ICA algorithm, $S'_{k,j}$ which is the part of data recovered by ICA satisfies the following relations:

$$\overline{S'_k} = \sum_{j=0}^{N-1} S'_{k,j}/N = 0, \quad (3)$$

$$\sum_{j=0}^{N-1} S'_{p,j} S'_{q,j}/N = \delta_{p,q}. \quad (4)$$

Here, by applying the centering process to the original sequences, namely $\overline{S_k} = 0$, the sequences recovered by ICA have orthogonality as

$$\sum_{j=0}^{N-1} S^{\mathrm{ICA}}_{p,j} S^{\mathrm{ICA}}_{q,j}/N = \delta_{p,q}. \quad (5)$$

We call these sequences the orthogonal ICA sequences. By using sequences which have the good correlation property as the original ones, we can obtain the sequences which realize orthogonality in the synchronous case and can be used in the asynchronous system. In the case where the synchronization is broken, the performance of the sequences recovered by ICA is almost the same as the original sequences because of almost the same waveform each other.

The sequences which have these properties can be realized by the other ICA algorithms, such as the Equivariant Adaptive Separation via Independence (EASI) [6], [9], [10]. This algorithm have simple parallel structure and can be implemented simply into the hardware [10].

Here, we use the Chebyshev chaotic spreading sequences as the original for the orthogonal ICA sequences. It is known that these Chebyshev sequences have good correlation property and arbitrary number of the sequences, whose code lengths are also arbitrary, can be obtained easily [11], [12]. The k-th sequence is defined as follows:

$$S_{k,j+1} = T_q(S_{kj}), \quad q \ge 2. \quad (6)$$

Here, $T_q(x)$ is the q-th order Chebyshev polynomial defined by $T_q(\cos\theta) = \cos(q\theta)$ and j is time or position in this code sequence. It is known that this Chebyshev map is ergodic and it has the ergodic invariant measure

$$\rho(x)dx = \frac{dx}{\pi\sqrt{1-x^2}} \quad (7)$$

and it satisfies the orthogonal relation

$$\int_{-1}^{1} T_i(x)T_j(x)\rho(x)dx = \delta_{i,j}\frac{1+\delta_{i,0}}{2}, \quad (8)$$

where $\delta_{i,j}$ is the Kronecker delta function. From the above facts these chaotic sequences generated from

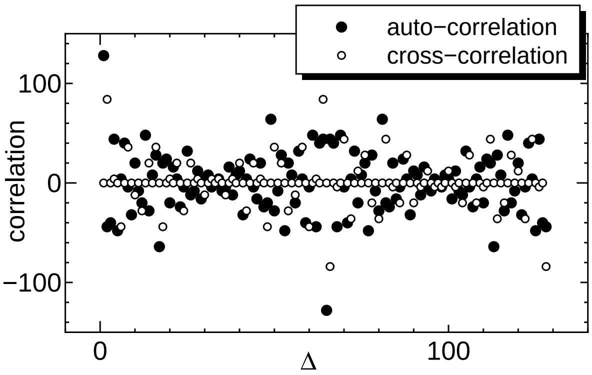

Fig. 1. Correlation property of the Walsh-Hadamard sequences.

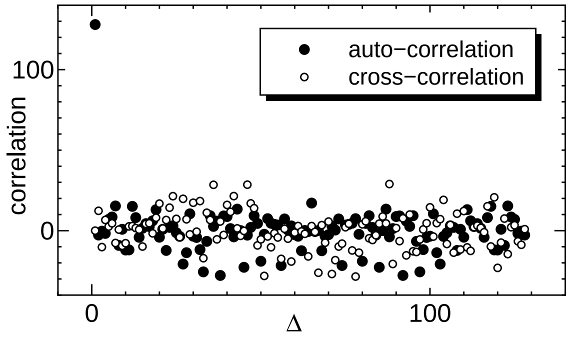

Fig. 2. Correlation property of the orthogonal ICA sequences whose originals are Chebyshev chaotic sequences.

$T_q(x)$ of different orders can be used as the naturally orthogonal spreading sequence in CDMA from the ergodic principle [11–14].

III. Correlation Property

We compare the correlation property of the orthogonal ICA sequences whose original sequences are the above Chebyshev chaotic sequences with that of the Walsh-Hadamard sequences. We calculate the auto-correlations and the cross-correlations given by $\sum_{j=0}^{N-1} S_{k,j} S_{k,j+\Delta}$ and $\sum_{j=0}^{N-1} S_{k,j} S_{i,j+\Delta} \ \ (i \neq k)$, respectively. Here, the sequence code satisfies the relation $S_{k,0} = S_{k,N}$ and Δ means the phase difference. Figures 1 and 2 show examples of correlation properties of the Walsh-Hadamard sequences and the orthogonal ICA sequences as functions of the phase difference Δ for these code length $N = 128$, respectively. From these figures, it is found that the orthogonal ICA sequences have better correlation properties to be synchronized with the received signal at the correlation receiver than the Walsh-Hadamard sequences.

IV. Bit Error Rate

We compare the performance of the band-limited baseband synchronous CDMA using the orthogonal ICA sequence with that using the Walsh-Hadamard sequence. For this, we calculate the BERs by numerical simulations.

In this study, we set the number of users for $K = 10$ and the spreading factor for $N = 128$. The average power of each chip of spreading sequence is set for unity 1. Thus, the initial spread bit energy E is equal to N. Here, we investigate the system where the transmitted signal powers after passing the filter are not controlled. Thus, the transmitted signal energy in the air, whose bandwidth is limited, is usually lower than E. This is because a filter for limiting the bandwidth cut a certain amount of bit energy. For limiting the bandwidth, we use the Raised Cosine filter and the bandwidth $(1 + \alpha)/T$ is set for $(1 + \alpha)T_{\rm c}$, where $1/T_{\rm c}$ is a chip rate. We consider the channel where the additive white Gaussian channel noise whose power is σ^2 exists. We obtain the distribution of the correlation output of the receiver by 10000 sample bits numerical simulation and calculate the mean value and the variance of this distribution . We use the randomly selected sequences from the set obtained in $N = 128$. By using these two values and the complementary error function, BERs are estimated. Finally, we can obtain the average BERs in the systems using the orthogonal ICA sequences or the Walsh-Hadamard sequences.

We also compare these BERs with those in band-unlimited synchronous CDMA. The quantity BER in the band-unlimited CDMA using the orthogonal spreading sequence is already known in [4]. The quantity in baseband system can be estimated by using the standard Gaussian approximation and Signal to Interference plus Noise Ratio (SINR) obtained analytically in [4]. Based on Pursley [15], we have derived the SINR of the orthogonal sequences in the band-unlimited baseband synchronous CDMA as

$$\mathrm{SINR} = \sqrt{\frac{E}{\sigma^2}}. \tag{9}$$

Figure 3 shows the BERs as functions of initial bit energy per noise power. Initial bit energy means the one before passing the RC filter. The line is drawn by using the above analytical SINR and the standard Gaussian approximation. Each point is the data obtained by numerical simulation. Table I shows the a part of the BER data in Fig. 3. These values are for 18 dB.

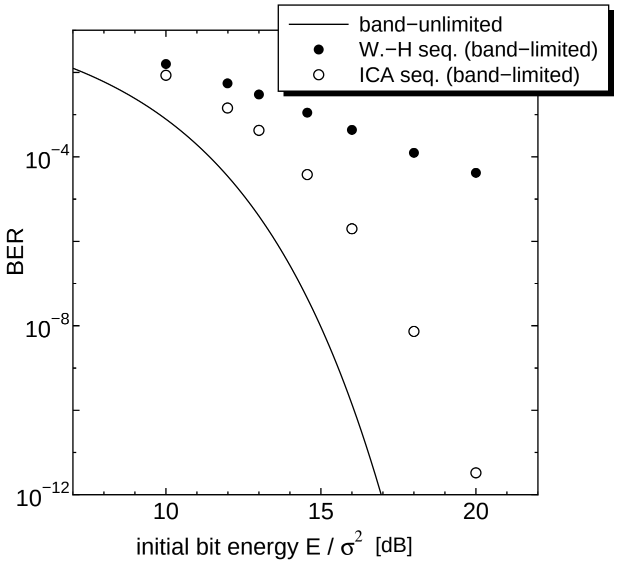

Fig. 3. BERs in synchronous CDMA using the several orthogonal sequences as functions of initial bit energy per noise power. The line is drawn by using the analytical formula and each point is obtained by numerical simulation for $K = 10$, $E = 128$.

Theoretical figure of BER in band-unlimited baseband system using the orthogonal sequences for the same parameters as the above simulation are $9.84449 * 10^{-16}$.

TABLE I
BERS IN BAND-LIMITED BASEBAND SYNCHRONOUS CDMA FOR ABOUT 18 dB.

Walsh-Hadamard seq.	$1.25026 * 10^{-4}$
Orthogonal ICA seq.	$7.36994 * 10^{-9}$

From these values and figure, it is found that the BERs in the band-limited baseband system are larger than that in band-unlimited baseband system. However, the BER in the band-limited system using the orthogonal ICA sequences is much closer to that in the band-unlimited system than that using the Walsh-Hadamard sequences.

V. DISCUSSION AND CONCLUSION

In the present paper, the performance of the band-limited baseband synchronous CDMA using the orthogonal ICA sequences is investigated and be compared with that using the Walsh-Hadamard sequences. Here, these are the systems where the transmitted signal powers after passing the filter are not controlled. We calculate the BER of these systems by numerical simulation. We focus on the relation between the BER and initial bit energy per noise power which is the one before passing the RC filter. In addition, these BERs in the band-limited synchronous CDMA are compared with those in the band-unlimited synchronous CDMA using orthogonal sequences. It is found that the BERs in the band-limited systems are larger than that in the band-unlimited baseband system. However, the BER in the band-limited baseband system using the orthogonal ICA sequences is much closer to the band-unlimited system than that using the Walsh-Hadamard sequences. The orthogonal ICA sequences can realize much lower BER than the Walsh-Hadamard sequences in the band-limited synchronous CDMA. This is because it is highly possible that the part of signal energy which are cut off by the Raised Cosine filter in the system using the orthogonal ICA sequences is much lower than that using the Walsh-Hadamard sequences. That is to say, it is highly possible that the spectrum density of the orthogonal ICA sequences inside the limited bandwidth is larger than that of the Walsh-Hadamard sequences. In addition, the orthogonality realized in the band-unlimited system is broken in the band-limited system.

For the band-unlimited synchronous CDMA, the Walsh-Hadamard sequence is one of the most effective spreading sequences. Thus, this sequence is used now in the actual system, namely the band-limited synchronous CDMA such as down-link, as the most effective spreading sequence based on the performance in the band-unlimited system. However, from these results, it is found that the orthogonal ICA sequences are more effective significantly for the band-limited synchronous CDMA than the Walsh-Hadamard sequences.

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REFERENCES

[1] A. Hyvärinen, J. Karhunen and E. Oja, ed, Independent Component Analysis, Wiley, 2001.

[2] K. Umeno, “Chaos and codes in communication systems”, *Proc. IPSJ Symposium*, vol.19, pp.312–316, 2005.

[3] R. Takahashi, S.J. Kim and K. Umeno, “Super amplification of SNR with an independent component analysis in chaos CDMA”, *Technical Report of IEICE*, vol. 106, no. 413, NLP2006–102, pp. 1–6, Dec. 2006.

[4] R. Takahashi, S.J. Kim and K. Umeno, “Analysis of bit error rate using the chaos spreading sequence with an independent component analysis filter”, *Technical Report of IEICE*, vol. 106, no. 573, NLP2006-142, pp. 1–6, March 2007.

[5] R. Takahashi, S.J. Kim and K. Umeno, "Performance of the recovered pseudo-noise binary sequences by independent component analysis for CDMA", *Proc. 2007 Int. Symposium on Nonlinear Theory and Its Applications (NOLTA2007)*, pp.148–151, Sept. 2007.
[6] R. Takahashi, S.J. Kim and K. Umeno, "Generating complete orthogonal spreading sequences by equivariant adaptive separation via independence for CDMA", *Proc. 15th IEEE Int. Workshop on Nonlinear Dynamics of Electronic Systems (NDES2007)*, pp.126–128, July, 2007.
[7] A. Hyvärinen, "Fast and robust fixed-point algorithms for independent component analysis", *IEEE Trans. Neural Netw.*, vol.10, pp.626–634, May 1999.
[8] Available: http://itpp.sourceforge.net/
[9] J.F. Cardoso and B.H. Laheld, "Equivariant adaptive source separation", *IEEE Trans. Signal Processing*, vol.44, pp.3017–3030, Dec. 1996.
[10] S.J. Kim, K. Umeno and R. Takahashi, "FPGA implementation of EASI algorithm", *IEICE Electron. Express*, vol.4, no.22, pp.707–711, 2007.
[11] K. Umeno and K. Kitayama, "Spreading sequences using periodic orbits of chaos for CDMA", *Electron. Lett.*, vol. 35, no. 7, pp. 545–546, 1999.
[12] K. Umeno and K. Kitayama, "Improvement of SNR with chaotic spreading sequences for CDMA", *Proc. 1999 IEEE Information Theory Workshop*, p. 106, 1999.
[13] K. Umeno and A. Yamaguchi, "Construction of optimal chaotic spreading sequence using Lebesgue spectrum filter", *IEICE Trans. Fundamentals*, vol.E85–A, no.4, pp.849–852, April 2002.
[14] C.C. Chen, K. Yao, K. Umeno and E. Biglieri, "Design of spread-spectrum sequences using chaotic dynamical systems and ergodic theory," *IEEE Trans. Circuits Syst. I*, vol. 48, pp. 1110–1114, Sep. 2001.
[15] M.B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication - Part I: System analysis", *IEEE Trans. Commun.*, vol. 25, pp. 795–799, 1977.

Ryo Takahashi received his BSc, MSc and PhD degrees in physics from Tokyo University of Science, Japan, in 2001, 2003 and 2006, respectively. He was an expert researcher at National Institute of Information and Communications Technology (NICT), Japan, and be currently a research scientist at RIKEN, Japan. His research interests include statistical physics, nonlinear dynamical systems, and information and communications theory. He is a member of the IEEE, the Physical Society of Japan and the Japan Society for Industrial and Applied Mathematics.

Ken Umeno received his BSc degree in electronic communication from Waseda University, Japan, in 1990. He received his MSc and PhD degrees in physics from University of Tokyo, Japan, in 1992 and 1995, respectively. From 1995 to 1998, he was a special postdoctoral fellow of RIKEN, Japan. He joined the National Institute of Information and Communications Technology (NICT), Japan, in 1998. He lead the Chaos-CDMA project team of Japan Science and Technology Corporation during the term 2000-2003. Currently, he is a CEO and president of ChaosWare Inc. and a senior researcher of NICT. His research interests include chaos-based computation and chaos-based communication networks as well as the fundamental theory of dynamical systems and computer science. He received LSI IP Award in 2003.