Self-organized optimization and synchronization of material flow networks with decentralized control

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Abstract— **The efficient and reliable operation of material flows in transportation networks is a subject of broad economic interest. Important applications include the control of signalized intersections in urban road systems and the planning and scheduling of logistic processes. Traditional approaches to operating material flow networks are however known to have severe disadvantages: centralized controllers suffer from their high computational demands that make an on-line control hardly possible in larger networks, whereas a decentralized control using clearing policies leads under rather general conditions to instabilities. As an alternative approach that may help to overcome these problems, a self-organization mechanism of conflicting flows is proposed that is inspired by oscillatory phenomena of pedestrian and animal flows at intersections or bottlenecks. For this purpose, a permeability function is introduced that allows to sequentially serve the different possible flow directions at an intersection in a fully demand-dependent way.**

The self-organized optimization achieved by the presented approach is demonstrated to be closely linked to synchronization of the oscillatory service dynamics at the different intersections in the network. For regular grid topologies, different synchronization regimes are present depending on the inertia of the switching from one service state to the next one. The dependence of this observation on the regularity of the considered network is tested. The reported results contribute to an improved understanding of the conditions that have to be present for efficiently operating material flow networks by a decentralized control, which is of major importance for future implementations in real-world traffic or production systems.

I. Introduction

Many real-world complex systems have (among others) the function of transportation of material and/or information from one place to the other. Examples include systems in technology (including vehicular traffic [1], [2], [3], production, logistics, supply networks [4], or telecommunication) as well as biology (for example, the nervous and cardio-vascular system, intracellular transport using the cytoskeleton [5], [6], and nutrient transport in amoeboid organisms [7] or fungal mycelia [8]). One may distinguish continuous-flow systems (for example, power grids, water supply networks, nutrient or blood transport systems in organisms) from such systems which are characterized by a large number of individual and mutually interacting transportation processes (which is the case for most information flow networks, road, railway, pedestrian or animal traffic, production and logistics systems).

In the case of systems characterized by discrete flows, the aim of an efficient organization is to minimize the time required for all individual transportation processes. Typically, this optimization is difficult and demanding, since the topology of the underlying networks is composed of a potentially large number of merges and intersections at which there are conflicts between the flows on different routes. To avoid physical collisions, these flows have to be controlled by devices like traffic lights. The operation strategy of these devices is decisive for the optimization of the system performance.

Whereas in the case of a low network load, the individual service of transportation units is beneficial, due to the necessary safety headways between individual services, it becomes inefficient if the traffic volume in a material flow network exceeds a certain threshold. Hence, in the presence of substantially high traffic volumes, a coordinated operation of the conflicting flows leads to better results. Such a coordinated operation is achieved by bundling material or vehicles into platoons, which is performed in urban road networks by the action of traffic lights, or in logistics by transporting heavy loads on railways instead of roads.

In a coordinated service of conflicting material flows, the switching between flows from and/or in different directions leads to an accumulation of material (like vehicles or products) on the links which are currently not served. The corresponding effects are mathematically described in terms of queueing theory. In switched queueing systems, every intersection of conflicting material flows is characterized by the amount of delayed material on all of the incoming links, which is determined by the lengths $N(t)$ of the associated queues. The arrival and departure rates of material, $A(t)$ and $O(t)$, are bounded by the maximum capacity $\hat{Q}$ that is an intrinsic property of the transportation route and the used devices. Regarding the state of the different queues (which determine the current service state of the intersection point), one may distinguish different states of the queue: a "no service" state and a service period, which itself is composed of a "setup" state, a "clearing" state, and a possible "extension" state with free-flow conditions on the served routes (see Fig. 1). In the context of vehicular traffic control, a "setup" state of duration τ is essential for a safe operation making sure that all vehicles

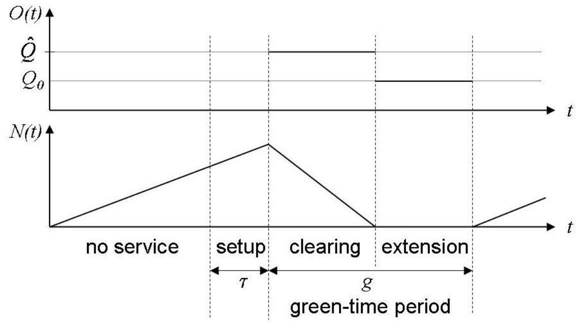

Fig. 1. Evolution of the departure flow (outflow) $O(t)$ and the amount of queued material (vehicles) $N(t)$ for one incoming link with a constant inflow rate $A(t) \equiv Q_0 < \hat{Q}$. The different states of the queue are schematically shown.

have left the conflict area before the considered traffic stream enters, whereas "clearing" and "extension" states combine to the total green time g during a service period.

Each of the states mentioned above is associated with different dynamical regimes of the queueing process, which are characterized by the relationship between the arrival flow and the change in the amount of queued material:

$$\frac{dN}{dt} = \begin{cases} A(t), & \text{“no service” and “setup” states,} \\ A(t) - \hat{Q}, & \text{“clearing” state,} \\ 0, & \text{“extension” state.} \end{cases} \tag{1}$$

In a similar way, one may write for the departure flow:

$$O(t) = \begin{cases} 0, & \text{“no service” and “setup” states,} \\ \hat{Q}, & \text{“clearing” state,} \\ A(t), & \text{“extension” state.} \end{cases} \tag{2}$$

Present day, material flow networks are typically subjected to a global control. Beside the high computational demands of such a control in the case of larger networks, the systems become very unflexible with respect to their reactions to random fluctuations of the transportation volume and extraordinary events like traffic accidents, machine failures, or attacks. Despite these disadvantages, for example, typical traffic lights are mainly operated by traffic-adaptive fixed-time controllers, which means that the average service period is fixed and can only be varied within small intervals [9]. The natural alternative of a decentralized control is very appealing in terms of computational demands and flexibility. However, the traditional approach to such a control using clearing policies has been shown to lead to severe instabilities in networks [10]: even if a corresponding policy is stable and optimal for a single intersection, it may not be used for controlling acyclic networks which are commonly present in traffic, production, and logistics. In this work, an alternative strategy is proposed for a decentralized control of conflicting material flows in networks, which is motivated by empirical observations of self-organized oscillations of pedestrian motion at bottlenecks [11]. Using the resulting concept of a oscillating permeability function that is controlled by the net pressure difference between the conflicting flows, the self-organized optimization and synchronization of the flows in a simple network configuration is studied in some detail.

This paper is organized as follows: In Sec. II, the basic ingredients of a model for an efficient decentralized control and its generalization to arbitrary network configurations are described. The resulting switching dynamics at an isolated merge or intersection of two flows is considered in Sec. III. If it is applied to regular grid networks, the proposed control leads to a system-wide coordination with a phase coherent, mutually lagged switching dynamics of the individual intersection, which is studied in some detail in Sec. IV. Finally, the major results of this study are summarized and completed by an outlook on possible modifications of the presented approach that appear to be necessary for a practical implementation in real-world material flow networks.

II. Description of the Model

The model behind the decentralized control strategy used in this work is motivated by empirical findings from the field of pedestrian dynamics. Suppose there are two flows of pedestrians in opposite directions which have to pass a common bottleneck. Using simulations with the social force model assuming interactions between individuals due to physical and "social" pressure terms, Helbing *et al.* [11], [12] have been able to explain the empirical observation of an oscillatory switching between different flow directions by a varying net pressure difference. Corresponding findings have also been experimentally reported for ant traffic [13]. In addition, related effects have been found both empirically and by means of simulations in terms of self-organized lane formation in intersecting pedestrian flows [14] and pedestrian counter-flows [15] as well as for intersecting and bottleneck flows of pedestrian and vehicular traffic [16], [17], [18].

The above mentioned results suggest that the oscillatory switching between traffic in different directions is induced by the variations of the mutual (physical as well as social) pressure of the waiting and moving pedestrians. Mathematically, this phenomenon can be formulated by a cost function $C(t)$ which controls the permeability of the bottleneck in both directions. For the sake of simplicity, this function will be firstly specified for the case of two merging or intersecting flows. In the presented model, there are two main factors entering the cost function [19]:

1) a net "pressure" force of the waiting material, which is proportional to the difference between the amounts of material still waiting to cross the intersection in both possible directions;
2) a net "drain" force that corresponds to the natural inertia of a flow that is currently served; it will be assumed that this drain is proportional to the difference of the departure flows in both directions.

As an additional factor, one may also penalize the total waiting time $T(t)$ of the delayed material waiting on the incoming links. In the pedestrian analogy, this additional factor may be reasonable as pedestrians waiting for a relatively long time become impatient and, as a consequence, enhance their pressure on the crowd with increasing waiting time. Summarizing, one may formulate the following cost function:

$$\begin{aligned} C(t) =& \alpha_N (N_2(t) - N_1(t)) \\ &+ \alpha_O (O_2(t) - O_1(t)) \\ &+ \alpha_T (T_2(t) - T_1(t)), \end{aligned} \tag{3}$$

where the indices $i = 1, 2$ correspond to the incoming links, and $\alpha_{N,O,T}$ are proper weights. For simplicity, in the following the specific choice $\alpha_T = 0$, $\alpha_N = 1$ and $\alpha_O = \alpha$ will be considered.

Having specified the cost function, the next step is to define a permeability function $\gamma_i(t)$ for the whole set of incoming links i at a given intersection point. In the formalism used in this work [19], the possible choice of this function will be restricted by some general properties: First, $\gamma_i(t)$ is a multiplicative "factor" entering the dynamic equation for the outflows $O_i(t)$, i.e., there is no outflow from link i if $\gamma_i(t) = 0$. Second, $\gamma_i(t)$ has to be normalized, that is, $\gamma_i(t) \leq 1$. Third, $\gamma_i(t)$ should be a function of the current value of the cost function $C(t)$ exclusively. Fourth, $\gamma_2(t) = 1.0 - \gamma_1(t)$, i.e., in the case of a simple merge or intersection of two flows, the permeability for both incoming directions have to add to one. With this setting, it is possible to fully describe a self-organized oscillatory switching between flows in different directions.

In the case of pedestrian motion where simultaneous flows in both directions (and, hence, collisions between different pedestrians) are still possible, a logistic function [19]

$$\gamma_{1,2}(t) = \frac{1}{1 + \beta e^{\pm \eta C(t)}} \tag{4}$$

can be used as a specification for $\gamma_i(t)$ (see Fig. 2). A similar continuous parametrization may be used for describing transport processes in biological systems like the cardio-vascular system or related biological transportation networks, where a simultaneous service of different directions is possible under the action of pressure gradients or incorportating diffusive processes between the different flows. In contrast to such situations, in technological systems such as vehicular traffic, material transport in production systems by automatically guided vehicles or conveyors, or baggage handling networks at airports, a continuous parametrization with $0 \leq \gamma_i(t) \leq 1$ would allow for potential collisions between objects transported in different directions. However, in the mentioned systems, such collisions have to be avoided to reduce the occurrence of accidents and the resulting damages of the transported objects. For this purpose, it is beneficial to replace the logistic function by a piecewise constant function that can only approach binary values (corresponding to "stop" and "go" commands for the respective flows) [20], [21], [22]. An example for such a function is shown in Fig. 2. As a particular advantage, this choice allows a sharp switching between both states. In order to avoid losses of efficiency due to finite times required for accelerating and decelerating, an additional hysteresis effect may be introduced into the model, which causes the service state to remain the same if the traffic conditions in the operated and stopped directions are comparable and thus introduces a preferred switching interval at given traffic conditions.

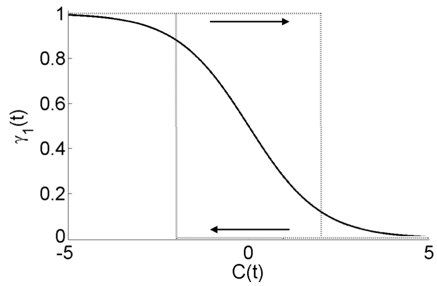

Fig. 2. Two specific parametrizations for the permeability function in dependence on the value of the cost $C(t)$. The solid line shows a logistic function (see Eq. (4)) that may be used for describing intersecting pedestrian or biological flows, whereas the dotted line corresponds to the hysterethic piecewise constant model used for controlling material flows in technological systems within the framework of this study.

In situations where the material flows can be operated in a two-phase mode (e.g., merges or intersections without turning conflicts), the resulting dynamics of the system is described by a small set of equations relating the amounts of waiting material $N_i(t)$, the arrival flows $A_i(t)$, the departure flows $O_i(t)$, and the permeability $\gamma_i(t)$ of the different directions (service phases) at the "intersection". In addition to the above considerations, one has to specify the relationship between the change of the queue lengths and the associated arrival and departure flows (which is rather trivial) and an explicit expression for the departure flows. Following the considerations from Sec. I, in the case of a complete permeability of the intersection in direction i (i.e., $\gamma_i(t) = 1$), the departure flow $O_i(t)$ from the respective queue occurs with maximum rate $\hat{Q}_i$ if there is material left in the queue, or otherwise with the arrival rate $A_i(t)$ of new material which is instantaneously processed. Summarizing, one finds the following set of equations:

$$\frac{d}{dt} N_i(t) = A_i(t) - O_i(t) \tag{5}$$

$$O_i(t) = \gamma_i(t) \times \begin{cases} A_i(t), & N_i(t) = 0 \\ \hat{Q}_i, & N_i(t) > 0 \end{cases} \tag{6}$$

$$\gamma_1(t) = \begin{cases} 1, & C(t) < -\Delta C/2, \\ 0, & C(t) > \Delta C/2 \end{cases} \tag{7}$$

$$\gamma_2(t) = 1 - \gamma_1(t) \tag{8}$$

$$C(t) = \left(\sum_{i \in \mathcal{L}_2} - \sum_{i \in \mathcal{L}_1} \right) (N_i(t) + \alpha O_i(t)), \tag{9}$$

where the permeability functions show a bistable behavior in the interval $[-\Delta C/2, \Delta C/2]$ of the cost function, $\mathcal{L}_s$ are the sets of compatible flows that can be served in a joint phase s, and $\hat{Q}_i$ are the maximally possible flows in the direction corresponding to the queue i. In order to give different priorities to different queues, it is also possible to use an asymmetric window $[C_{min}, C_{max}]$ (or, under more general conditions, route-specific thresholds C_i) for defining the switching thresholds of the permeability function. Note that the above set of equations neglects finite setup times τ required for a safe operation of real-world material flows. However, such setup times may be easily incorporated into the model, such that it is general enough to describe network flows in a variety of situations, including urban road traffic, transportation of goods in factories (e.g., by automatically guided vehicles), logistics, biological systems, or even the routing of data packages in certain kinds of information networks.

If the operation of a node in a material flow network requires more than two disjoint service phases (for example, in case of intersections in urban road networks where left-hand turning needs to be taken into account), the above formalism can be generalized in a straightforward way [22], yielding a control of the dynamics by assigning a priority index

$$P_s(t) := \sum_{i \in \mathcal{L}_s} \left(N_i(t) + \alpha O_i(t) \right) \tag{10}$$

to each service phase $\mathcal{L}_s$, operating the phase with the highest priority index until the cost function

$$C_s(t) := P_s(t) - \frac{1}{S-1} \sum_{s' \neq s} P_{s'}(t). \tag{11}$$

falls below a certain threshold, and then switching to the next phase. Again, in order to avoid extraordinarily high waiting times on connections with a low load, it is possible to additionally penalize the total waiting time $T_i(t)$ of the material stored on each link by an additional term proportional to $T_i(t)$ in the definition of the priority index.

III. Dynamics of Isolated Intersections

In the case of an isolated merge or turning-free intersection of two conflicting material flows, the main properties of the switching dynamics can be analyzed analytically. In order to have stable flow conditions without a successive congestion of any of the links, the average outflow $\overline{O}_i$ over one service period must equal the average inflow $\overline{A}_i$ on every link. Assuming the link capacity being the same on every incoming link ($\hat{Q}_i = \hat{Q}$), there are only two dynamically relevant parameters remaining: the total intersection load defined as $u = (A_1 + A_2)/\hat{Q}$, and the inflow ratio $r := A_2/A_1$ [22].

A. Constant Inflows

If the corresponding arrival flows $A_i(t)$ are assumed to be constant with sub-critical values (that is, the sum of all inflows is beyond the capacity limit of the intersection[1], it is possible to derive exact expressions for the switching periods, the duration of eventual extension periods, and the minimum and maximum amounts of material stored on the different transportation routes.

[1]In general, $u < 1 - f(\tau))$ where f is a monotonously increasing function of the incorporated setup time τ, which is neglected in the considered model.

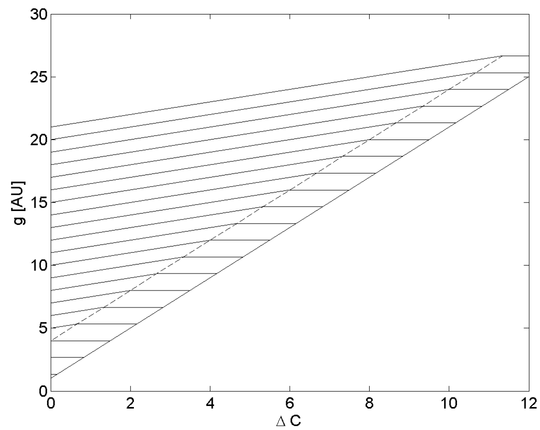

Fig. 3. Dependence of the green time g (in arbitrary units (AU)) on the switching threshold ΔC for $A_1 = A_2 = 0.25$ and $\alpha = 1.0$ in the regimes of incomplete clearing, complete clearing, and complete clearing with extension phase. While the second queue j is assumed to be completely cleared at the beginning of the service period, the different lines correspond to initial queue lengths of $N_i = 20, 19, 18, \dots$ (from top to bottom). The dashed line indicated the transition between incomplete and complete clearing.

Using a piecewise constant permeability function as described in Sec. II with equal switching thresholds $\Delta C/2$ for all possible service states, one may easily convince oneself that for an initial queue length N_i^0, a complete clearing requires a time

$$T_{C,i} = \frac{N_i^0}{\hat{Q}_i - A_i}. \tag{12}$$

However, this complete clearing takes only place if the choice of the switching threshold $\Delta C/2$ is large enough, in particular,

$$\Delta C \geq 2 \left[\frac{A_j N_i^0}{\hat{Q}_i - A_i} + N_j^0 - \alpha \hat{Q}_i \right]. \tag{13}$$

Otherwise, the switching to a service of the remaining link j occurs already after a time

$$T_{I,i} = \frac{\alpha \hat{Q}_i + N_i^0 - N_j^0 + \Delta C/2}{\hat{Q}_i - (A_i - A_j)}. \tag{14}$$

If this threshold is sufficiently large to allow a complete clearing of the queue i, there may be eventually another extension phase with a duration of

$$T_{E,i} = \frac{\alpha A_i - N_j(T_{C,i}) + \Delta C/2}{A_j} \tag{15}$$

with $N_j(T_{C_i}) = N_j^0 + A_j T_{C,i}$. Following these arguments, in all cases the duration of the service period for link i, T_i, increases linearly with the switching threshold $\Delta C/2$. Hence, ΔC may be thought of as a characteristic scale determining the cycle time T of the traffic light for fixed traffic volumes described by the incoming flows A_1 and A_2.

Fig. 3 illustrates the above analytical findings for two symmetric inflows with a road utilization of 0.5. In the dependence of the green time on the switching threshold, two regimes can be distinguished: complete clearing and extension (lowest line with a constant slope) and incomplete clearing (shifted lines with smaller slope). Both regimes are separated by a region where a complete clearing of the queue without an extension phase takes place. The width of this transitional regime is determined by the value of α, and the corresponding time increases linearly with the initial queue length N_i^0. For small values of N_i^0 (here: $N_i^0 < 4$, there is always a complete clearing of the queue within one service period due to the drain force realized by the parameter α. In contrast to this, for larger initial queues, the green time required for complete clearing increases linearly with both switching threshold ΔC and initial queue length N_i^0 as expected.

The above results suggest that for sufficiently large ΔC, extension phases can be found on both incoming links for a wide range of arrival rates (see [22]). In general, the switching dynamics of an isolated intersection can be qualitatively classified by considering whether the service period of one or both queues leads to an incomplete (I) or complete (C) clearing and even an eventual extension (E) phase. The decision which of the possible combinations (II, IC, IE, CC, CE, or EE) is realized is determined by the inflows A_1 and A_2 (or, alternatively, the quantities u and r), the switching threshold $\Delta C/2$ of the permeability function, and the weight α quantifying the impact of the net drain. Previous investigations [22] have revealed that concerning the qualitative switching dynamics, there are two striking features: First, the presence of an incomplete clearing requires a rather low traffic volume on the considered link, whereas there is much more traffic on the second one (i.e., for low values of r and high values of u). Such a phase can only affect the less frequently used link. Second, extension states can be found at almost all traffic conditions, except of such with a sufficiently high intersection load u. Moreover, as one would expect, there is a strong correlation between long green times and the presence of extension phases. Note, however, that the presence of an extension phase must not necessarily correspond to optimal traffic conditions in terms of usage of the total node capacity, which is related to the freedom in defining the switching threshold ΔC.

Summarizing, the switching threshold parameter ΔC determines (together with the arrival flow rates and initial queue lengths) the typical durations of a service period g in terms of a monotonously increasing piecewise linear function. In addition, the parameter α weighting the influence of the net drain due to the current departure flows is responsible for a complete clearing of the queue within a finite interval of ΔC. Choosing parameter combinations within this range, the self-organized control strategy leads to an instantaneous switching to the next service period as the queue has been completely cleared. If $\alpha \to 0$, the switching period is exclusively determined by the net "pressure" difference $N_2(t) - N_1(t)$, such that for arbitrary initial conditions, there is no guarantee that any of the involved queues will be completely cleared. If in contrast $\alpha \gg 1$, the drain will dominate the dynamics, which means that a switching can only take place when the presently served queue is empty. From this perspective, this case has a strong similarity to one of the standard clearing policies, "clear largest buffer" (CLB).

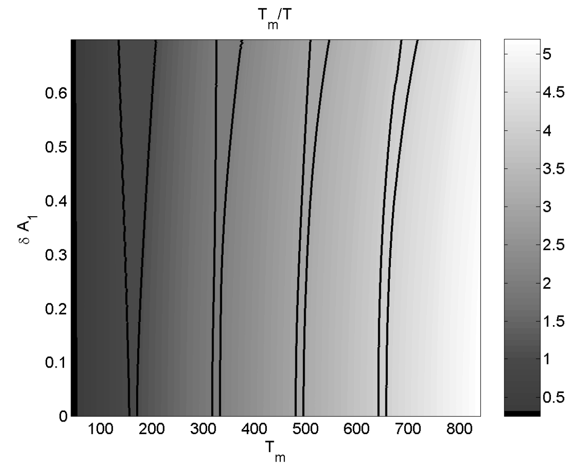

Fig. 4. Emergence of phase-locked states in the case of a periodic arrival flow $A_1(t) = \langle A_1 \rangle \left(1 + \delta A_1 \sin(2\pi t/T_m)\right)$ on link 1 and a constant arrival flow on link 2. The results are shown for arrival flows with $\langle A_1 \rangle = A_2 = 0.25$ (all values are normalized with respect to the link capacity $\hat{Q}$), $\Delta C = 20$ and $\alpha = 1.0$. Gray-scale colors correspond to the ratio between arrival flow period T_m and switching period T, the black lines indicate parameter regions where the deviation from a perfect 1:1 to 4:1 frequency locking (from left to right) is smaller than 5%.

B. Periodic Inflows

The case of constant arrival flows of material discussed above is rather artificial. For example, in an urban road network, vehicles are usually bundled to platoons by the action of traffic lights. Since these traffic lights operate in a periodic way, the outflow on a link i and, hence, the inflow to the queue at the downstream end of this link ia described by a periodic function, too. If the arrival flows are determined by a periodic function with period T_m, for some distinct interval of these periods (which depends on ΔC), the switching period T of the self-organized control locks to this external demand period [20]. If the amplitude of the periodic arrival flow increases, the width of this locking window increases as well. Moreover, there are other windows of higher-order frequency locking with a similar behavior, which are indicated in Fig. 4. The detailed position and width of the corresponding Arnold tongues is determined by the parameters α and ΔC of the permeability function $\gamma_i(t)$ and the average inflows $\langle A_1 \rangle$ and $\langle A_2 \rangle$. In general, the choice of these parameters again yields a naturally preferred switching frequency in the case of constant inflows, whose existence gives rise to the non-trivial locking intervals.

It has to be mentioned that the existence of various phase-locked states can be observed independently of the shape of the periodic functions. In particular, under certain situations, non-trivial $m{:}n$ locking states can be found [20]. One may speculate that the appearance and width of such states can be enhanced by different modifications of the scenario studied in this work, for example, a) by decreasing the critical switching threshold ΔC (i.e., allowing for a complete clearing of all queues with extension phases under almost all conditions) or adjusting it in a way that the preferred switching time is close to the modulation period, b) by changing from "positive" to "negative" hysteresis in the permeability function (i.e., choosing a *negative* value of ΔC), which would correspond to an anticipative switching regime, c) by choosing continuous instead of piecewise constant permeability functions, or d) by considering periodic arrival flows with more pronounced temporal variability profiles (for example, arrival flows described by rectangular (on-off) functions instead of sinusoidals). It should be noted that options b) and c) have been used in a recent study [20], revealing a much larger variety of locking windows than in the scenario shown in Fig. 4. A detailed examination of the corresponding dynamic effects is however outside the scope of the presented study.

IV. Dynamics of Coupled Intersections

Whereas the dynamics of isolated intersections can be (under certain assumptions) treated analytically, the practically more relevant case of controlling networks of intersecting flows is more challenging. In the following, this problem is hence addressed by means of simulations.

A. Specification of the Scenario

In order to keep the number of variables as low as possible, in the following, some simplifications will be made:

1) For avoiding an unlimited congestion of the incoming links, which would finally affect the whole network, it is required that the sum of the maximum inflows for all necessary service phases of a given intersection is sufficiently smaller than the maximum link capacity $\hat{Q}$, which is assumed to be equal for all links.
2) The studied networks consist only of nodes of degree $k = 4$ (i.e., with four pairs of ingoing and outcoming links which connect neighboring pairs of nodes). A consideration of nodes with $k = 3$ (i.e., a merge or diverge) is also possible, however, the case of $k > 4$ is not considered here. All links are assumed to allow a bi-directional traffic.
3) Motivated by the problem of vehicular traffic in networks with right-hand driving policy, for every intersection and every incoming link, only right-hand turning is permitted with a given probability $p \in [0, 1]$. In contrast to this, direct left-hand turning will remain forbidden, as the corresponding possibility would call for two additional turning phases at least if the arrival flows are sufficiently large. Moreover, in the case of road networks with bi-directional traffic, left-hand turning may be effectively achieved by a sequence of right-hand turnings.

Under these assumptions, the cycle of successive service periods consists of only two phases. The dynamic coupling of different intersection points in a material flow network requires to represent the inflows at a given node by the outflows from neighboring nodes at an earlier time. In a zeroth-order approximation, the corresponding time delay is assumed to equal the free-flow travel time τ_i^{free} on the respective link,

$$A_i(t) = \sum_{j \neq i} \alpha_{ji} O_j(t - \tau_i^{free}), \tag{16}$$

where α_{ji} is the fraction of the flow on link j which is turning to link i ($\alpha_{ji} = p$ for right-hand turning, $\alpha_{ji} = 0$ for left-hand-turning, and $\alpha_{ji} = 1 - p$ otherwise). As a necessary condition of material conservation, $\sum_i \alpha_{ji} = 1$ for all links j. To approach more realistic conditions, the free-flow travel time has to be replaced by a load-dependent travel time, i.e., a time-dependent travel time which is determined by the amount of material waiting on the link. A very simple way for doing this would be setting

$$\tau_i(t) = \frac{l_i - N_i(t)\Delta l}{v_{free}}, \tag{17}$$

where l_i is the total length of the road between two neighboring intersections and Δl the space occupied by one of the queued objects.

Using the described coupling between neighboring nodes, the switching dynamics has been studied for regular grid networks with 25 nodes where the material flows are operated the same way (i.e., with the same parameters of the permeability function) at all intersections. In a simplified setting without load-dependent travel times and left-hand turning, it has been observed that a network-wide self-organization of the flows takes place that leads to a certain minimization of the total amount of waiting material [21]. As one increases the preferred switching threshold determined by the parameter ΔC, the total capacity of the network, but also the amount of delayed material successively increase.

B. Phase Coherence Analysis

In order to better understand the dynamics of the self-organization process due to the proposed decentralized control, a detailed phase coherence analysis has been performed. For evaluating the presence of phase coherence in this context, the appropriate definition of a monotonously increasing phase variable is necessary. Without loss of generality, the initial phase of node j has been defined in a way that $\phi_j = 0$ corresponds to the time of the first switching of its permeability function. In a similar way, $\phi_j = (n - 1)\pi$ then corresponds to the time of the n-th switching at this node. Between these switching times, the phase variable $\phi(t)$ is defined by linear interpolation. Although this definition leads to an increase of the phase which may be periodically modulated if the "on" and "off" times for one specific direction are not symmetric, in the long-term limit, these variables may be used for a phase coherence analysis.

In order to quantify the phase coherence in a multivariate way, different approaches based on the mean resultant length

$$r_{jk} = \left| \left\langle e^{i\Delta\phi_{jk}(t)} \right\rangle_t \right| = \frac{1}{T} \left| \sum_{t=1}^{T} e^{i\Delta\phi_{jk}(t)} \right| \tag{18}$$

(with $\Delta\phi_{jk}(t) = \phi_k(t) - \phi_j(t)$) as a particularly useful bivariate measure for phase coherence have been considered [21]: a) global or neighbor-based averages of this pairwise phase coherence index, b) average mean resultant length of all intersections with respect to their mean-field, c) the synchronization cluster strength computed using the synchronization cluster analysis algorithm of Allefeld and Kurths [23], d) eigenvalue statistics obtained from the matrix of pairwise indices in terms of the generalized synchronization cluster analysis [24], and e) the LVD dimension density method [20], [21] which describes the average exponential scaling of the residual variances obtained from the eigenvalues.

Whereas all mentioned measures yield comparable results for model systems like a network of Kuramoto phase oscillators with long-range interactions [21], they show a quantitatively different behavior when applied to the switching dynamics in the considered material flow model, which is most likely explained by their different sensitivity to heterogeneity effects. In particular, the global and neighbor-based averages and the average coherence with the mean-field quantify only the mean degree of phase coherence, but are not sensitive to detect effects of spatial heterogeneity. The synchronization cluster strength assumes implicitly the presence of a unique synchronization cluster[2], which is under general conditions a too strong assumption, especially during the transition from non-coherent to phase-coherent dynamics. The number and average strength of synchronization clusters based on the generalized synchronization cluster analysis allow a better characterization of heterogeneity effects, but yield rather coarse measures. Finally, the phase coherence parameter based on the LVD dimension density approach is well suited to quantify heterogeneities, but is only a coarse and uncertain measure for the strength of phase synchronization. In summary, a combined consideration of different approaches is here helpful to distinguish information about the average phase coherence and its heterogeneity.

Looking at the dynamics of the material flow model in some more detail, it turns out that the probability $\alpha_{ji} = p$ of right-hand turning serves as a coupling parameter between neighboring intersections (at least as long as possible sources or sinks along the links of the network are neglected): If p is large, only a lower amount of material crossing one intersection arrives also the next one, which corresponds to a low coupling of the dynamics, and vice versa. In a two-parameter study in dependence on both p and the switching threshold ΔC, multivariate phase coherence analysis reveals pronounced Arnold tongues that correspond to different phase coherent regimes in the system (see Fig. 5). These tongues are separated by parameter regions that correspond to an incoherent switching at the different nodes. The presence of multiple

[2]Note that the measures used here for quantifying phase coherence have been introduced in the context of phase synchronization analysis, while the notation of phase synchronization is doubtful in the context considered here since the intersecting material flows subjected to decentralized control do not represent self-sustained oscillators, which are a prerequisite for phase synchronization in the standard definition [25].

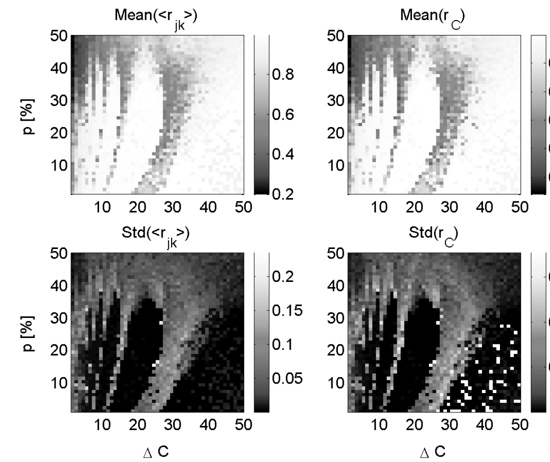

Fig. 5. Mean values (upper panels) and standard deviations (lower panels) of the mean pairwise mean resultant length $\langle r_{jk} \rangle$ (left) and synchronization cluster strength r_C (right), obtained from different parts of a set of long simulation runs for a regular 5×5 grid network with constant travel times $\tau = 30$s, fixed randomly chosen inflows A_i on the links entering from outside the network, and $\alpha = 0.1$. Whereas in the phase coherent regimes, this coherence is found to be stable in time (low standard deviations), in the incoherent parts, the considered measures fluctuate also significantly in time (high standard deviations). Note that the results concerning the synchronization cluster strength r_C may be less reliable, as the corresponding algorithm does not necessarily converge correctly in all cases.

regimes appears to be a consequence of the presence of two disjoint time scales in the dynamics: the preferred switching period described by the threshold ΔC in the permeability functions $\gamma_i(t)$, and the free-flow travel times τ_i^{free} that are equal for all links in the considered grid-like topology.

C. Synchronization Analysis

In the previous paragraphs, evidence has been reported for different regimes with a coherent switching at different nodes. As the notation of phase synchronization cannot be used in this context without criticism due to the missing of well-defined self-sustained oscillators, it is of interest how the above findings may be interpreted in a synchronization context. In order to avoid systematic errors in the estimation of phase coherence indices due to the piecewise linear phase definition applied in the context of switching dynamics, it may be beneficial to use alternative synchronicity concepts instead that refer to the isochronicity (or fixed-lag synchronicity) of switching events. Similar approaches are already used for quantifying synchronization of events, for example, in neurophysiological systems.

Consider the switching dynamics of two individual intersections in the network, which consist of two distinct phases under the assumptions described above. In the case of a phase coherent dynamics, the cycle length at all intersections must be the same. Then, for every pair of oscillations, the following quantities are constructed:

- For the second last switching event at intersection j, the switching event at intersection k is identified which has a minimum time lag with respect to this event. This time lag

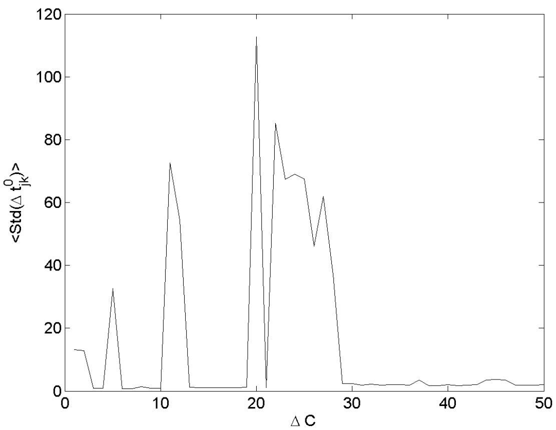

Fig. 6. Mean standard deviations of the switching-time differences at different intersections as a function of the switching threshold parameter ΔC for $\alpha = 0.1$ and $p = 0.05$.

is computed. After this, the sequences of switching events are traced backwards in time for both intersections event-by-event, leading to a series of time differences $\Delta t_{jk}^{(0)}$.

- In a similar way, a sequence of time differences $\Delta t_{jk}^{(1)}$ is constructed where the reference event at intersection k is replace by its predecessor.

In the case of (almost) isochronous switching, the values of $\Delta t_{jk}^{(0)}$ will be centered around zero with a narrow distribution. If the switching is synchronous with a certain lag, $\Delta t_{jk}^{(0)}$ and/or $\Delta t_{jk}^{(1)}$ are characterized by a small standard deviation with non-zero mean (depending on the symmetry of the switching dynamics). Following this idea, the mean value of $\Delta t_{jk}^{(0)}$ can be used as a measure of isochronicity, whereas the minimum of the standard deviations of both sequences yields a measure for the presence of (lagged) event synchronicity, and the corresponding mean value determines the average lag. The concept of event synchronization analysis can be further refined, however, this is outside of the scope of the presented work. For example, in the case of delays covering more than one switching cycle, an optimal lag could be inferred by considering the standard deviations of all event sequences that are shifted with respect to each other by $\pm k$ events and identifying the minimum of the resulting function.

The results of this event synchronization analysis (see Fig. 6) are in good qualitative agreement with those obtained from the phase coherence analysis [21]. In particular, for a fixed turning probability p, there are distinct values of the switching threshold for which the standard deviations of the mutual event-time differences between different intersections approach values near zero, indicating a possibly lagged synchronization of the switching.

D. Heterogeneity Effects

The structural properties of the system studied so far have been fully homogeneous in space. In order to understand the impact of corresponding heterogeneities to the coherent switching dynamics and, hence, the optimal self-organization of the network, it has been studied how distributed travel times corresponding to a deviation from the regular grid structure influence the degree of synchronization in the switching dynamics of the network. In a recent paper [21], it has been demonstrated that the measures of phase coherence discussed above decay only slowly as the disorder of the system increases. In general, this disorder is reflected in an increasing amount of delayed material and a systematic tendency towards shorter service intervals. From this, one may argue that the presence of heterogeneities limits the capability of material flow systems to optimally self-organize in terms of the local control strategy used in this work.

It is likely that also an uneven spatial distribution of other parameters (in particular, turning probabilities, switching thresholds, or the mutual weights of the different force terms entering the cost function $C(t)$) will lead to similar consequences. Of more fundamental interest, however, is the question how a consideration of dynamically adjusted travel times incorporating queued road sections as well as acceleration and deceleration effects may affect the resulting material flow dynamics. This question will be further addressed in future studies.

V. Conclusions

In many socio-economic systems, in particular in the fields of production and logistics, there has already been a paradigm shift from a centralized control towards a decentralized self-organization of material flows [26]. In the special case of urban traffic networks, the situation is however still quite different, as local control strategies have so far not been very successful in practical implementations. A particular reason for this is that "traditional" decentralized control approaches based on clearing policies may lead to unstable traffic conditions due to the presence of dynamic feedbacks. The results reported in this contribution may be an important step in overcoming these problems. However, for specific practical applications, there may be a need for incorporating additional mechanisms for stabilizing the large-scale dynamics, for example, by including demand anticipation over a finite time horizon [9], [27] or by implementing additional heuristic strategies for avoiding local deadlocks [9], [28].

In the presented work, a general concept has been discussed for serving conflicting material flows in general networks in a fully demand-dependent way. Apart from the potential applicability of this approach to controlling material flows in real-world networks, it may also be used for modelling and understanding flows in a variety of other (in particular, biological) systems. The key ingredient of a permeability function which is determined by gradient forces like net pressures or drains can be specified in such systems for describing discrete as well as continuous flows.

It has been shown that an appropriate specification of the permeability function leads to a fully self-organized and synchronized dynamics of the traffic at intersections, which is significantly more flexible than traditional approaches using a centrally enforced cyclic traffic light control. Hence, using the presented approach for implementing a decentralized control strategy at all intersections of a material flow network, the time-delayed local coupling between the flows at neighboring nodes leads to the emergence of a network-wide phase coherent switching dynamics that can be understood in the context of event synchronization. As a particularly interesting feature, the presence of multiple disjoint synchronization regimes in networks has been revealed, which is a so far not yet fully understood dynamical phenomenon possibly related to the presence of two intrinsic time scales of the dynamics, corresponding to the preferred switching intervals prescribed by the parameter ΔC of the considered model and the typical travel times τ_i on the different links.

Although in the presented considerations, a continuous flow approximation has been used for describing all material flows, theoretical investigations [29] as well as simulations not discussed here in detail suggest that the resulting dynamics on the network is very similar if the individual transportation units or agents are explicitly considered. Moreover, in this study, a variety of simplifications have been made, which are necessary for a detailed analytical treatment of the dynamical properties of the permeability model. Additional effects have to be explicitly taken into account in future studies, including the influence of spatial heterogeneity and distributed parameters, the dynamical feedback between queue lengths and travel times, the definition of setup phases, acceleration effects leading to a delayed clearing of queues, etc. It will be of particular interest how these effects influence the emergence of a synchronized switching and, as a consequence, the efficiency of a self-organized traffic light control within the network.

The question how the observed self-organization processes are related to an optimization of network flows has not been discussed in much detail in this paper. From the perspective of temporal variability, synchronization introducing regularity of material flows on large spatial scales can be considered as a particular optimization goal. However, regarding the minimization of throughput (or, alternatively, waiting) times in the network, one has to recall that the duration of service periods increases monotonously with the switching threshold ΔC. However, when tuning this parameter in a way that corresponds to a transition from a non-synchronous to a synchronous switching, the corresponding increase of delayed material is significantly reduced. These preliminary results have so far only been verified for switching intervals without setup times. Recent results in the field of urban traffic networks [28] however suggest that the transition to a synchronized, not necessarily phase-coherent service could lead to a significant decrease in the total waiting times. The further validation of this hypothesis within a more general framework of arbitrary material flow networks will be an important topic of future research.

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